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Telltale traces of U(1) fields in noncommutative standard model extensions

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Abstract: Restrictions imposed by gauge invariance in noncommutative spaces together with the effects of ultraviolet/infrared mixing lead to strong constraints on possible candidates for a noncommutative extension of the Standard Model. In this paper, we study a general class of 4-dimensional noncommutative models consistent with these restrictions. Specifically we consider models based upon a gauge theory with the gauge group $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \ldots \times \mathrm{U}(N_m)$ coupled to matter fields transforming in the (anti)-fundamental, bi-fundamental and adjoint representations. Noncommutativity is introduced using the Weyl-Moyal star-product approach on a continuous space-time. We pay particular attention to overall trace-U(1) factors of the gauge group which are affected by the ultraviolet/infrared mixing. We show that, in general, these trace-U(1) gauge fields do not decouple sufficiently fast in the infrared, and lead to sizable Lorentz symmetry violating effects in the low-energy effective theory. Making these effects unobservable in the class of models we consider would require pushing the constraint on the noncommutativity mass scale far beyond the Planck mass ($M_{\mathrm{NC}} \gtrsim 10^{100}\, M_{\mathrm{P}}$) and severely limits the phenomenological prospects of such models.

Keywords: Non-Commutative Geometry, Renormalization Group, Beyond Standard Model.

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Contents

1. Introduction and discussion of results

Gauge theories on spaces with noncommuting coordinates

$$[x^\mu, x^\nu] = i\,\theta^{\mu\nu} \ , \tag{1.1}$$

provide a very interesting new class of quantum field theories with intriguing and sometimes unexpected features. These noncommutative models can arise naturally as low-energy effective theories from string theory and D-branes. As field theories they must satisfy a number of restrictive constraints detailed below, and this makes them particularly interesting and challenging for purposes of particle physics model building. For general reviews of noncommutative gauge theories the reader can consult e.g. refs. [1 – 3].

There are two distinct approaches used in the recent literature for constructing quantum field theories on noncommutative spaces. The first approach uses the Weyl-Moyal star-products to introduce noncommutativity. In this case, noncommutative field theories are defined by replacing the ordinary products of all fields in the Lagrangians of their commutative counterparts by the star-products

$$(\phi * \varphi)(x) \equiv \phi(x)\ e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}}\ \varphi(x) \ . \tag{1.2}$$

Noncommutative theories in the Weyl-Moyal formalism can be viewed as field theories on ordinary commutative spacetime. For example, the noncommutative pure gauge theory action is

$$S = -\frac{1}{2g^2}\int d^4x\ \ \mathrm{Tr}(F_{\mu\nu} * F^{\mu\nu}) \ , \tag{1.3}$$

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where the commutator in the field strength also contains the star-product. The important feature of this approach is the fact that phase factors in the star-products are not expanded in powers of θ and the θ dependence in the Lagrangian is captured entirely. This ability to work to all orders in θ famously gives rise to the ultraviolet/infrared (UV/IR) mixing [4, 5] in the noncommutative quantum field theory which we will review below.

The second approach to noncommutativity does not employ star-products. It instead relies [6, 7] on the Seiberg-Witten map which represents noncommutative fields as a function of θ and ordinary commutative fields. This approach essentially reduces noncommutativity to an introduction of an infinite set of higher-dimensional (irrelevant) operators, each suppressed by the corresponding power of θ, into the action. There are two main differences compared to the Weyl-Moyal approach. First, in practice one always works with the first few terms in the power series in θ and in this setting the UV/IR mixing cannot be captured. Second, the Seiberg-Witten map is a non-linear field transformation. Therefore, one expects a non-trivial Jacobian and possibly a quantum theory different from the one obtained in the Weyl-Moyal approach. In the rest of this paper we will concentrate on the Weyl-Moyal approach.

In the context of Weyl-Moyal noncommutative Standard Model building, a number of features of noncommutative gauge theories have to be taken into account which are believed to be generic [8]:

1. the mixing of ultraviolet and infrared effects [4, 5] and the asymptotic decoupling of U(1) degrees of freedom [9, 10] in the infrared;

2. the gauge groups are restricted to U(N) groups [11, 12] or products thereof;

3. fields can transform only in (anti-)fundamental, bi-fundamental and adjoint representations [13 – 15];

4. the charges of matter fields are restricted [16] to 0 and ± 1, thus requiring extra care in order to give fractional electric charges to the quarks;

5. gauge anomalies cannot be cancelled in a chiral noncommutative theory [13, 16 – 21], hence the anomaly-free gauge theory must be vector-like.

Building upon an earlier proposal by Chaichian et al. [22], the authors of ref. [8] constructed an example of a noncommutative embedding of the Standard Model with the purpose to satisfy all the requirements listed above. The model of [8] is based on the gauge group U(4) $\times$ U(3) $\times$ U(2) with matter fields transforming in noncommutatively allowed representations. Higgs fields break the noncommutative gauge group down to a low-energy commutative gauge theory which includes the Standard Model group SU(3) $\times$ SU(2) $\times$ U(1)$_Y$. The U(1)$_Y$ group here corresponds to ordinary QED, or more precisely to the hypercharge Y Abelian gauge theory. The generator of U(1)$_Y$ was constructed from a linear combination of *traceless* diagonal generators of the microscopic

theory U(4) × U(3) × U(2). Because of this, the UV/IR effects – which can affect only the overall trace-U(1) subgroup of each U(N) — were not contributing to the hypercharge U(1)$_Y$. However some of the overall trace-U(1) degrees of freedom can survive the Higgs mechanism and thus contribute to the low-energy effective theory, in addition to the Standard Model fields. These additional trace-U(1) gauge fields logarithmically decouple from the low-energy effective theory and were neglected in the analysis of ref. [8]. The main goal of the present paper is to take these effects into account.

We will find that the noncommutative model building constraints, and, specifically, the UV/IR mixing effects in the trace-U(1) factors in the item 1 above, lead to an unacceptable defective behavior of the low-energy theory, when we try to construct a model having the photon as the only massless colourless U(1) gauge boson. Our findings rule out a class of noncommutative extensions of the Standard Model.

(a) This class is based on a noncommutative quantum gauge theory defined on a four-dimensional continuous space-time (UV cutoff sent to infinity). Within the Weyl-Moyal approach there are two ways to avoid our conclusions. Either one can introduce extra dimensions [23] or one can give up the continuous space-time.

(b) Noncommutative models we concentrate on are similar to the example in [8] and should be distinguished from earlier ones studied in [22] for two reasons.[1] First, we include the effects of the UV/IR mixing in our analysis. Second, is that our models preserve full noncommutative gauge invariance including the Higgs and Yukawa sectors. As such, the difficulties related to unitarity violation discussed in [25] do not apply in our case.

(c) Finally, as already mentioned earlier, we are not pursuing the Seiberg-Witten map approach and as such our conclusions cannot be directly applied to the class of noncommutative models which rely on Taylor expansion in powers of θ in [7, 6, 26 – 32].

The UV/IR mixing in noncommutative theories arises from the fact that certain classes of Feynman diagrams acquire factors of the form $e^{ik_\mu\theta^{\mu\nu}p_\nu}$ (where k is an external momentum and p is a loop momentum) compared to their commutative counter-parts. These factors directly follow from the use of the Weyl-Moyal star-product (1.2). At large values of the loop momentum p, the oscillations of $e^{ik_\mu\theta^{\mu\nu}p_\nu}$ improve the convergence of the loop integrals. However, as the external momentum vanishes, $k \to 0$, the divergence reappears and what would have been a UV divergence is now reinterpreted as an IR divergence instead. This phenomenon of UV/IR mixing is specific to noncommutative theories and does not occur in the commutative settings where the physics of high energy degrees of freedom does not affect the physics at low energies.

[1] The construction in [8] of correct values of hypercharges of the Standard Model from the product gauge group was influenced by [22]. The authors of ref. [22] advocated a noncommutative model which satisfied criteria 2, 3 and 4 listed in the beginning of this section. Their model was based on the noncommutative gauge group U(3) × U(2) × U(1) with matter fields transforming only in (bi-)fundamental representations, and remarkably, it predicted correctly the hypercharges of the Standard Model. In many respects their model is similar to the bottom-up approach of [24] to the string embedding of the Standard Model in purely commutative settings. Unfortunately, the noncommutative U(3) × U(2) × U(1) model of [22] ignores all the effects of the UV/IR mixing which alters the infrared behavior of the U(1) hypercharge sector.

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There are two important points concerning the UV/IR mixing [5, 9, 10, 12] which we want to stress here. First, the UV/IR mixing occurs only in the trace-U(1) components of the noncommutative U(N) theory, leaving the SU(N) degrees of freedom unaffected. Second, there are two separate sources of the UV/IR mixing contributing to the dispersion relation of the trace-U(1) gauge fields: the Π_1 effects and the Π_2 effects, as will be explained momentarily.

A study of the Wilsonian effective action, obtained by integrating out the high-energy degrees of freedom using the background field method, and keeping track of the UV/IR mixing effects, has given strong hints in favour of a non-universality in the infrared [9, 10]. In particular, the polarisation tensor of the gauge bosons in a noncommutative U(N) gauge theory takes form [5, 9, 10]

$$\Pi^{AB}_{\mu\nu} = \Pi^{AB}_1(k^2,\tilde{k}^2)\,\left(k^2 g_{\mu\nu} - k_\mu k_\nu\right) + \Pi^{AB}_2(k^2,\tilde{k}^2)\,\frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2}\,, \qquad \text{with } \tilde{k}_\mu = \theta_{\mu\nu}k^\nu\,. \tag{1.4}$$

Here $A, B = 0, 1, \ldots N^2-1$ are adjoint labels of U(N) gauge fields, A^A_μ, such that $A, B = 0$ correspond to the overall U(1) subgroup, i.e. to the trace-U(1) factor. The term in (1.4) proportional to $\tilde{k}_\mu\tilde{k}_\nu/\tilde{k}^2$ would not appear in ordinary commutative theories. It is transverse, but not Lorentz invariant, as it explicitly depends on $\theta_{\mu\nu}$. Nevertheless it is perfectly allowed in noncommutative theories. It is known that Π_2 vanishes for supersymmetric noncommutative gauge theories with unbroken supersymmetry, as was first discussed in [5].

In general, both Π_1 and Π_2 terms in (1.4) are affected by the UV/IR mixing. More precisely, as already mentioned earlier, the UV/IR mixing affects specifically the $\Pi^{0\,0}_1$ components and generates the $\Pi^{0\,0}_2$ components in (1.4). The UV/IR mixing in $\Pi^{0\,0}_1$ affects the running of the trace-U(1) coupling constant in the infrared,

$$\frac{1}{g(k,\tilde{k})^2_{\mathrm{U}(1)}} = 4\Pi^{0\,0}_1(k^2,\tilde{k}^2) \;\to\; -\frac{b_0}{(4\pi)^2}\,\log k^2\;, \qquad \text{as } k^2 \to 0\,, \tag{1.5}$$

leading to a logarithmic decoupling of the trace-U(1) gauge fields from the SU(N) low-energy theory, see refs. [8 – 10] for more detail.

For nonsupersymmetric theories, $\Pi^{0\,0}_2$ can present more serious problems. In theories without supersymmetry, $\Pi^{0\,0}_2 \sim 1/\tilde{k}^2$, at small momenta, and this leads to unacceptable quadratic IR singularities [5]. In theories with softly broken supersymmetry (i.e. with matching number of bosonic and fermionic degrees of freedom) the quadratic singularities in $\Pi^{0\,0}_2$ cancel [5, 9, 10]. However, the subleading contribution $\Pi^{0\,0}_2 \sim const$, survives [33] unless the supersymmetry is exact. For the rest of the paper we will concentrate on noncommutative Standard Model candidates with softly broken supersymmetry, in order to avoid quadratic IR divergencies. In this case, $\Pi^{0\,0}_2 \sim \Delta M^2_{\mathrm{susy}}$,[2] as explained in [33]. The presence of such Π_2 effects will lead to unacceptable pathologies such as Lorentz-noninvariant dispersion relations giving mass to only one of the polarisations of the trace-U(1) gauge field, leaving the other polarisation massless.

The presence of the UV/IR effects in the trace-U(1) factors makes it pretty clear that a simple noncommutative U(1) theory taken on its own has nothing to do with ordinary QED.

[2] $\Delta M^2_{\mathrm{SUSY}} = \frac{1}{2}\sum_s M^2_s - \sum_f M^2_f$ is a measure of SUSY breaking.

The low-energy theory emerging from the noncommutative U(1) theory will become free at $k^2 \to 0$ (rather than just weakly coupled) and in addition will have other pathologies [8 – 10, 33]. However, one would expect that it is conceivable to embed a commutative SU(N) theory, such as e.g. QCD or the weak sector of the Standard Model into a supersymmetric noncommutative theory in the UV, but some extra care should be taken with the QED U(1) sector [8]. We will show that the only realistic way to embed QED into noncommutative settings is to recover the electromagnetic U(1) from a *traceless* diagonal generator of some higher U(N) gauge theory. So it seems that in order to embed QED into a noncommutative theory one should learn how to embed the whole Standard Model [8]. We will see, however, that the additional trace-U(1) factors remaining from the noncommutative U(N) groups will make the resulting low-energy theories unviable (at least for the general class of models considered in this paper).

In order to proceed we would like to disentangle the mass-effects due to the Higgs mechanism from the mass-effects due to non-vanishing Π_2. Hence we first set $\Pi_2 = 0$ (this can be achieved by starting with an exactly supersymmetric theory). It is then straightforward to show (see section 4) that the Higgs mechanism alone cannot remove all of the trace-U(1) factors from the massless theory. More precisely, the following statement is true: *Consider a scenario where a set of fundamental, bifundamental and adjoint Higgs fields breaks* $U(N_1) \times U(N_2) \times \cdots \times U(N_m) \to H$, *such that* H *is non-trivial. Then there is at least one generator of the unbroken subgroup* H *with non-vanishing trace. This generator can be chosen such that it generates a U(1) subgroup.*

We can now count all the massless U(1) factors in a generic noncommutative theory with $\Pi_2 = 0$ and after the Higgs symmetry breaking. In general we can have the following scenarios for massless U(1) degrees of freedom in H:

(a) $\mathrm{U}(1)_Y$ is traceless and in addition there is one or more factors of trace-U(1) in H.

(b) $\mathrm{U}(1)_Y$ arises from a mixture of traceless and trace-U(1) generators of the noncommutative product group $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \cdots \times \mathrm{U}(N_m)$.

(c) $\mathrm{U}(1)_Y$ has an admixture of trace-U(1) generators as in (b) plus there are additional massless trace-U(1) factors in H.

In the following sections we will see that none of these options lead to an acceptable low-energy theory once we have switched on $\Pi_2 \neq 0$, i.e. once we have introduced mass differences between superpartners. It is well-known [5, 33] that $\Pi_2 \neq 0$ leads to strong Lorentz symmetry violating effects in the dispersion relation of the corresponding trace-U(1) vector bosons, and in particular, to mass-difference of their helicity components. If option (a) was realised in nature, it would lead (in addition to the standard photon) to a new colourless vector field with one polarisation being massless, and one massive due to Π_2.

The options (b) and (c) are also not viable since an admixture of the trace-U(1) generators to the photon would also perversely affect photon polarisations and make some of them massive.[3]

[3]One could hope that the trace-U(1) factors could be made massive at the string scale by working in a

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In the rest of the paper we will explain these observations in more detail.

We end this section with some general comments on noncommutative Standard Modelling. This paper refines the earlier analysis of [8]. In that work the trace-U(1) factors were assumed to be completely decoupled in the extreme infrared and, hence, were neglected. However, it is important to keep in mind that the decoupling of the trace-U(1)'s is logarithmic and hence slow. Even in presence of a huge hierarchy between the noncommutative mass scale $M_{\rm NC}$, say of the order of the Planck scale $M_{\rm P} \sim 10^{19}$ GeV, and the scale $\Lambda \sim (10^{-14} - 10^{9})\,\mathrm{eV}$ (electroweak and QCD scale, respectively), where the SU(N) subgroup becomes strong, the ratio

$$\frac{g^2_{\mathrm{U}(1)}}{g^2_{\mathrm{SU}(N)}} \sim \frac{\log\left(\frac{k^2}{\Lambda^2}\right)}{\log\left(\frac{M^4_{\rm NC}}{\Lambda^2 k^2}\right)} \gtrsim 10^{-3} \tag{1.6}$$

is not negligible. In particular, the above inequality holds for any $M_{\rm NC} > k \gtrsim 2\Lambda$. Hence the complete decoupling of the trace-U(1) degrees of freedom at small non-zero momenta does not appear to be fully justified and the trace-U(1) would leave its traces in scattering experiments at accessible momentum scales $k \sim 1\,\mathrm{eV} - 10^{10}\,\mathrm{eV}$ (see section 2 for more detail).

2. UV/IR mixing and properties of the trace-U(1)

UV/IR mixing manifests itself only in the trace-U(1) part of the full noncommutative U(N). For this part it strongly affects Π_1 and is responsible for the generation of nonvanishing Π_2 (if SUSY is not exact). In this section we will briefly review how the UV/IR mixing arises in the trace-U(1) sector and how this leads us to rule out options (a) and (c) discussed in section 1.

2.1 Running gauge coupling

Following refs. [9, 10], we will consider a U(N) noncommutative theory with matter fields transforming in the adjoint and fundamental representations of the gauge group. We use the background field method, decomposing the gauge field $A_\mu = B_\mu + N_\mu$ into a background field B_μ and a fluctuating quantum field N_μ, and the appropriate background version of Feynman gauge, to determine the effective action $S_{\rm eff}(B)$ by functionally integrating over the fluctuating fields.

theory where these factors are anomalous. Then one could use the Green-Schwarz mechanism [34] to cancel the anomaly and simultaneously give a large stringy mass to these U(1) factors. This scenario which is often appealed to in ordinary commutative theories to remove unwanted U(1) factors cannot be used in the noncommutative setting. The reason is that at scales above the noncommutative mass, the noncommutative gauge invariance requires the gauge group to be U(N). It cannot become just an SU(N) theory (above the noncommutative scale) and remain noncommutative, see e.g. [21]. Therefore we require vector-like theories as stated in item 5.

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To determine the effective gauge coupling in the background field method, it suffices to study the terms quadratic in the background field. In the effective action these take the following form (capital letters denote full U(N) indices and run from 0 to N^2-1),[4]

$$S_{\text{eff}} \ni 2\int \frac{d^4k}{(2\pi)^4} B_\mu^A(k) B_\nu^B(-k) \Pi_{\mu\nu}^{AB}(k). \tag{2.1}$$

At tree level, $\Pi_{\mu\nu}^{AB} = (k^2 g_{\mu\nu} - k_\mu k_\nu)\,\delta^{AB}/g_0^2$ is the standard transverse tensor originating from the gauge kinetic term. In a commutative theory, gauge and Lorentz invariance restrict the Lorentz structure to be identical to the one of the tree level term. In noncommutative theories, Lorentz invariance is violated by θ. The most general allowed structure is then given by eq. (1.4). The second term may lead to the strong Lorentz violation mentioned in the introduction. This term is absent in supersymmetric theories [5, 9].

Let us start with a discussion of the effects noncommutativity has on Π_1 and the running of the gauge coupling. That is, for the moment, we postpone the study of Π_2-effects by considering a model with unbroken supersymmetry.[5] As usual, we define the running gauge coupling as

$$\left(\frac{1}{g^2}\right)^{AB} = \left(\frac{1}{g_0^2}\right)^{AB} + 4\Pi_{1\,\text{loop}}^{AB}(k). \tag{2.2}$$

where g_0^2 is the microscopic coupling (i.e. the tree level contribution) and Π_{loop} includes only the contributions from loop diagrams. Henceforth, we will drop the loop subscript.

To evaluate Π at one loop order one has to evaluate the appropriate Feynman diagrams. The effects of noncommutativity appear via additional phase factors $\sim \exp(i\frac{p\tilde{k}}{2})$ in the loop-integrals. Using trigonometric relations one can group the integrals into terms where these factors combine to unity, the so called planar parts, and those where they yield $\sim \cos(p\tilde{k})$, the so called non-planar parts.

For fields in the fundamental representation, the phase factors cancel exactly[6] and only the planar part is non-vanishing. Fundamental fields therefore contribute as in the commutative theory [9]. In all loop integrals[7] involving adjoint fields one finds the following factor [10],

$$M^{AB}(k,p) = \left(-d\sin\frac{k\tilde{p}}{2} + f\cos\frac{k\tilde{p}}{2}\right)^{ALM}\left(d\sin\frac{k\tilde{p}}{2} + f\cos\frac{k\tilde{p}}{2}\right)^{BML}. \tag{2.3}$$

Using trigonometric and group theoretic relations this collapses to

$$M^{AB}(k,p) = -N\;\delta^{AB}(1-\delta_{0A}\cos k\tilde{p}). \tag{2.4}$$

[4] We use euclidean momenta when appropriate and the analytic continuation when considering the equations of motion in subsection 2.2.

[5] Nevertheless, we will give general expressions for Π_1 valid also in the non-supersymmetric case.

[6] One may roughly imagine that for each fundamental field that appears in a Feynman diagram there is also the complex conjugate field which cancels the exponential factor.

[7] To keep the equations simple we consider in this section a situation where all particles of a given spin and representation have equal diagonal masses. Please note that the masses for fermions and bosons in the same representation may be different as required for SUSY breaking.

j=	scalar	Weyl fermion	gauge boson	ghost
α_j	-1	$\frac{1}{2}$	$-\frac{1}{2}$	1
C_j	0	$\frac{1}{2}$	2	0
d_j	1	2	4	1

Table 1: Coefficients appearing in the evaluation of the loop diagrams.

We can now easily see that all effects from UV/IR mixing, marked by the presence of the $\cos k\tilde{p}$, appear only in the trace-U(1) part of the gauge group. The planar parts, however, are equal for the U(1) and SU(N) parts.

Summing everything up we find the planar contribution (the coefficients α_j, C_j, d_j are given in table 1 and $C(\mathbf{r})$ is the Casimir operator in the representation $\mathbf{r}$)

$$\Pi_{1\,\text{planar}}(k^2) = -\frac{2}{(4\pi)^2}\Bigg(\sum_{j,\mathbf{r}} \alpha_j C(\mathbf{r})\Bigg[2C_j + \frac{8}{9}d_j + \int_0^1 dx\,\left(C_j - (1-2x)^2 d_j\right)\,\log\frac{A(k^2,x,m_{j,\mathbf{r}}^2)}{\Lambda^2}\Bigg]\Bigg), \tag{2.5}$$

where $m_{j,\mathbf{r}}$ is the mass of a spin j particle belonging to the representation $\mathbf{r}$ of the gauge group,

$$A(k^2,x,m_{j,\mathbf{r}}^2) = k^2 x(1-x) + m_{j,\mathbf{r}}^2, \tag{2.6}$$

and Λ appears via dimensional transmutation similar to $\Lambda_{\overline{\text{MS}}}$ in QCD. We have chosen the renormalisation scheme, i.e. the finite constants, such that $\Pi_{1\,\text{planar}}$ vanishes at $k = \Lambda$.

For the trace-U(1) part the nonplanar parts do not vanish and we find

$$\Pi_{1\,\text{nonplanar}} = \frac{1}{2k^2}\left(\hat{\Pi} - \tilde{\Pi}\right), \tag{2.7}$$

with

$$\hat{\Pi} = \frac{C(\mathbf{G})}{(4\pi)^2}\left\{\frac{8d_j}{\tilde{k}^2} - k^2\left[12C_j - d_j\right]\int_0^1 dx\; K_0(\sqrt{A}|\tilde{k}|)\right\}\quad, \tag{2.8}$$

$$\tilde{\Pi} = \frac{4C(\mathbf{G})}{(4\pi)^2}\left\{\frac{d_j}{\tilde{k}^2} - \left(C_j k^2 - d_j\frac{\partial^2}{\partial^2|\tilde{k}|}\right)\int_0^1 dx\; K_0(\sqrt{A}|\tilde{k}|)\right\}, \tag{2.9}$$

where $C(\mathbf{G}) = N$ is the Casimir operator in the adjoint representation.

For illustration, we plot in figure 1 the coupling (2.2) for a toy model which is a supersymmetric U(2) gauge theory with two matter multiplets and all masses (of all fields) taken to be equal. We observe that even for large masses the running of the U(1) part (solid lines) does not stop in the infrared. For masses smaller than the noncommutative mass scale $m^2 \ll M_{\text{NC}}$ the trace-U(1) gauge coupling has a sharp bend at M_{NC} where the nonplanar parts start to contribute. For larger masses the running stops at the mass scale m^2 only to resume running at a scale $\sim M_{\text{NC}}^4/m^2$ which is, of course, again due to the nonplanar parts. The dashed lines in figure 1 give the running of the SU(2) part which receives no nonplanar contributions and behaves like in an ordinary commutative theory. For $m^2 = 0$ the SU(2) gauge coupling reaches a Landau pole at $k = \Lambda$, for all non vanishing

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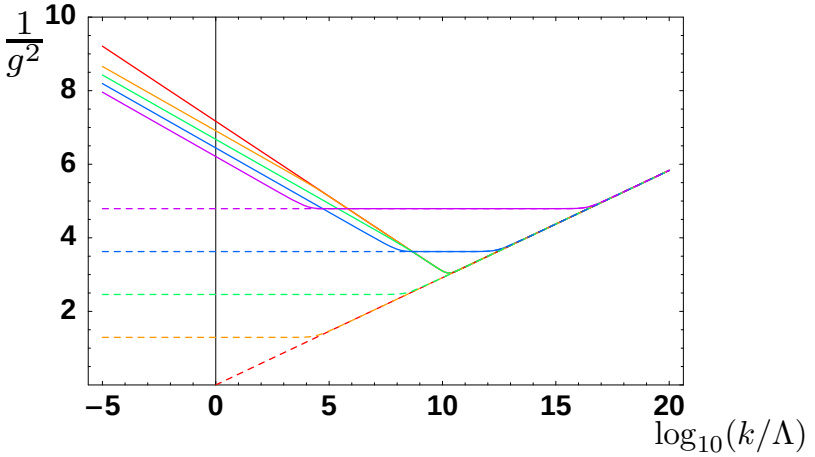

Figure 1: The running gauge couplings $g_{\mathrm{U}(1)}$ (solid) and $g_{\mathrm{SU}(2)}$ (dashed) for a U(2) theory with two matter multiplets and all particles of equal mass $m = 0, 10^4, 10^8, 10^{12}, 10^{16}\,\Lambda$, from top to bottom (left side, solid), as a function of the momentum k, for a choice of $|\tilde{k}| = \theta_{\mathrm{eff}}|k|$, with $\theta_{\mathrm{eff}} = 10^{-20}\Lambda^{-2}$.

masses the running stops at the mass scale. We observe that the ratio between the SU(2) coupling and the trace-U(1) coupling is not incredibly small over a wide range of scales, in support of our assertion (1.6) in section 1.

Further support comes from looking at the following approximate form for the running of the gauge coupling. We assume the hierarchy $\Lambda^2 \ll m^2 \ll M_{\mathrm{NC}}^2$,

$$\begin{aligned}
\frac{4\pi^2}{g_{\mathrm{U}(1)}^2} &= b_0^{\mathrm{p}} \log\left(\frac{k^2}{\Lambda^2}\right), && \text{for} \quad k^2 \gg M_{\mathrm{NC}}^2, \qquad (2.10)\\
\frac{4\pi^2}{g_{\mathrm{U}(1)}^2} &= b_0^{\mathrm{p}} \log\left(\frac{k^2}{\Lambda^2}\right) - b_0^{\mathrm{np}} \log\left(\frac{k^2}{M_{\mathrm{NC}}^2}\right), && \text{for} \quad m^2 \ll k^2 \ll M_{\mathrm{NC}}^2,\\
\frac{4\pi^2}{g_{\mathrm{U}(1)}^2} &= b_0^{\mathrm{p}} \log\left(\frac{m^2}{\Lambda^2}\right) - b_0^{\mathrm{np}} \left[\log\left(\frac{m^2}{M_{\mathrm{NC}}^2}\right) + \frac{1}{2}\log\left(\frac{k^2}{m^2}\right)\right], && \text{for} \quad k^2 \ll m^2.
\end{aligned}$$

The gauge coupling for the SU(N) subgroup $g_{\mathrm{SU}(N)}^2$ is obtained by setting $b_0^{\mathrm{np}} = 0$. For simplicity let us now consider a situation where we have only fields in the adjoint representation. One finds [8, 10] that $b_0^{\mathrm{np}} = 2b_0^{\mathrm{p}}$, and

$$\begin{aligned}
\frac{g_{\mathrm{U}(1)}^2}{g_{\mathrm{SU}(N)}^2} &= 1, && \text{for} \quad k^2 \gg M_{\mathrm{NC}}^2, \qquad (2.11)\\
\frac{g_{\mathrm{U}(1)}^2}{g_{\mathrm{SU}(N)}^2} &= \frac{\log\left(\frac{k^2}{\Lambda^2}\right)}{\log\left(\frac{M_{\mathrm{NC}}^4}{\Lambda^2 k^2}\right)}, && \text{for} \quad m^2 \ll k^2 \ll M_{\mathrm{NC}}^2,\\
\frac{g_{\mathrm{U}(1)}^2}{g_{\mathrm{SU}(N)}^2} &= \frac{\log\left(\frac{m^2}{\Lambda^2}\right)}{\log\left(\frac{M_{\mathrm{NC}}^4}{\Lambda^2 k^2}\right)}, && \text{for} \quad k^2 \ll m^2.
\end{aligned}$$

To reach

$$\frac{g^2_{\mathrm{U}(1)}}{g^2_{\mathrm{SU}(N)}} < \epsilon = 10^{-3} \tag{2.12}$$

we need $\log\left(\frac{M^4_{\mathrm{NC}}}{\Lambda^2 k^2}\right)$ and in turn M_{NC} to be large.

As a generic example let us use $\Lambda = \Lambda_W \sim 10^{-14}\,\mathrm{eV}$ (the scale where the ordinary electroweak SU(2) would become strong, in absence of electroweak symmetry breaking) and $k = 1\,\mathrm{eV}$.[8] We find

$$M_{\mathrm{NC}} > \Lambda^{\frac{1}{2}} k^{\frac{1}{2}} \exp\left(\frac{1}{4\epsilon}\log\left(\frac{k^2}{\Lambda^2}\right)\right) \sim 10^{6974}\, M_{\mathrm{P}}. \tag{2.13}$$

Taking electroweak symmetry breaking into account we have to replace $\log\left(\frac{k^2}{\Lambda^2}\right)$ by $\log\left(\frac{M^2_{\mathrm{EW}}}{\Lambda^2}\right)$ with $M_{\mathrm{EW}} \sim 100\,\mathrm{GeV}$ in (2.13). We find

$$M_{\mathrm{NC}} > 10^{12474}\, M_{\mathrm{P}}. \tag{2.14}$$

Let us increase the coupling strength of the SU(N) by using $\Lambda = 0.5\,\mathrm{eV}$. $k = 1\,\mathrm{eV}$ is now quite close to the strong coupling scale of the SU(N). Without symmetry breaking we find

$$M_{\mathrm{NC}} > 10^{131}\, M_{\mathrm{P}}. \tag{2.15}$$

We might be able to reduce this number by some orders of magnitude but without using an extreme field content it remains always incredibly large. Indeed, one can typically find a scale k which is not too close to the strong coupling scale of the SU(N) which strengthens the bounds dramatically. Therefore, as a conservative estimate we propose[9]

$$M_{\mathrm{NC}} > 10^{100}\, M_{\mathrm{P}}. \tag{2.16}$$

To conclude this subsection, let us point out that, in a scattering experiment (as depicted in figure 2), k is really the scale of the internal momentum, and therefore, non-vanishing. $\tilde{k}$, too, is non-vanishing in appropriate (remember that we have Lorentz symmery violation) directions of t-channel scattering.

2.2 The effects of a non vanishing Π_2 from SUSY breaking

In the previous subsection we made Π_2 vanish by working in a supersymmetric theory. Let us now study, what happens, when supersymmetry is (softly) broken.

Looking only at the trace-U(1) degrees of freedom of a generic noncommutative theory we have

$$\Pi_2 = \sum_j \alpha_j \left[\frac{1}{2}(3\tilde{\Pi}_j - \hat{\Pi}_j)\right]. \tag{2.17}$$

[8] It is obvious that $k^2 \ll M^2_{\mathrm{NC}}$. In this regime our formulas (2.10) and (2.11) approximate the full result to a very high precision since threshold effects are negligible.

[9] Of course, this constraint should not be taken overly serious. Above the string scale one should perform a string theory analysis. The main point is that the scale we find is way beyond the Planck scale.

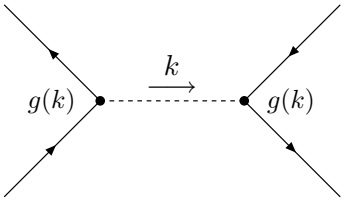

Figure 2: A typical Feynman diagram for scattering. The effective coupling g depends on the momentum k.

One easily checks that

$$\Pi_2 \sim \sum_j \alpha_j d_j f(k^2, \tilde{k}^2, m_j). \tag{2.18}$$

If SUSY is unbroken, all masses are equal. Using supersymmetric matching between bosonic and fermionic degrees of freedom,

$$\sum_j \alpha_j d_j = 0, \tag{2.19}$$

we reproduce the vanishing of Π_2. If SUSY is softly broken this cancellation is not complete anymore (in fact (2.19) still holds and this removes the leading power-like IR divergence in Π_2, however, the subleading effects in Π_2 survive). Π_2 gets a contribution [33]

$$\begin{aligned}\Pi_2 =& D \sum_j \alpha_j d_j m_j^2 \left[K_0(m\tilde{k}) + K_2(m\tilde{k}) \right] + O(k^2) \\ =& C \Delta M^2_{\mathrm{SUSY}} + C' \sum_j \alpha_j d_j m_j^2 \log(m_j^2 \tilde{k}^2) + \cdots,\end{aligned} \tag{2.20}$$

with known constants C, C' and D. This has dire consequences for the gauge boson. Let us look at the equations of motion resulting from this additional Lorentz symmetry violating contribution to the polarisation tensor (we briefly review the equations of motion for ordinary photons in Appendix A).

In presence of a Higgs field which generates a mass term m^2 and using unitary gauge the field equations in presence of non vanishing Π_2 read

$$\left(\Pi_1 (k^2 g_{\mu\nu} - k_\mu k_\nu) + \Pi_2 \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2} - m^2 g_{\mu\nu} \right) A^\nu = 0. \tag{2.21}$$

Using that unitary gauge implies Lorentz gauge, $k_\mu A^\mu = 0$, we can simplify

$$(\Pi_1 k^2 - m^2) A_\mu + \Pi_2 \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2} A^\nu = 0. \tag{2.22}$$

To proceed further it is useful to specify a direction for the momentum and the noncommutativity parameters. The photon flies in 3-direction and we have

$$k^\mu = (k^0, 0, 0, k^3). \tag{2.23}$$

What is the corresponding value of $\tilde{k}$? Since $\theta^{\mu\nu}$ breaks Lorentz invariance, we need to specify $\theta^{\mu\nu}$ in a particular frame. For the latter, a natural one is the system where the cosmic microwave background is at rest. In this frame, we assume that the only non-vanishing components of $\theta^{\mu\nu}$ are

$$\theta^{13} = -\theta^{31} = \theta. \tag{2.24}$$

This yields,

$$\tilde{k}_\mu = \theta_{\mu\nu}k^\nu = (0, \theta k^3, 0, 0), \quad \tilde{k}^{\,2} = (\theta k^3)^2. \tag{2.25}$$

We start with the ordinary transverse components of A^ν,

$$A_1^\nu = (0, 1, 0, 0). \tag{2.26}$$

In this direction, (2.22) yields

$$(\Pi_1 k^2 - m^2 - \Pi_2)A_{1,\nu} = 0. \tag{2.27}$$

In the other transverse direction,

$$A_2^\mu = (0, 0, 1, 0), \tag{2.28}$$

we find

$$(\Pi_1 k^2 - m^2)A_{2,\nu}. \tag{2.29}$$

Finally we have the third polarisation (which can be gauged away if and only if $m^2 = 0$),

$$A^\mu = (a, 0, 0, b), \quad k^0 a - k^3 b = 0 \tag{2.30}$$

which results in

$$(\Pi_1 k^2 - m^2)A_{3,\nu}. \tag{2.31}$$

We note that the different polarisation states do not mix due to the presence of Π_2. The second and the third polarisation state behave more or less like in the ordinary commutative case. However, the first has a modified equation of motion, (2.27), in presence of a non-vanishing Π_2.[10]

This is another strong argument against a trace-U(1) being the photon [33]. If the gauge symmetry is unbroken and $m^2 = 0$ we usually have two massless polarisations. However, a non vanishing Π_2 reduces this to one. The other one gets an additional mass Π_2. Since only one polarisation is affected this is a strong Lorentz symmetry violating effect. Moreover, a negative Π_2 would lead to tachyons while a positive mass is phenomenologically ruled out by the constraint [35]

$$m_\gamma < 6 \times 10^{-17}\,\mathrm{eV} \tag{2.32}$$

[10] One might argue that instead of eq. (2.27) one has to use the rescaled equation (we set $m^2 = 0$ for simplicity) $k^2 - \frac{\Pi_2(k^2,\tilde{k}^2)}{\Pi_1(k^2,k^2)} = 0$. For $k^2 \to 0$, the second term vanishes since Π_1 diverges in this limit. Therefore, we find an additional solution. However, this solution is rather strange. It does not correspond to a pole in the propagator (it goes like a log). Moreover, if one calculates the cross section Π_2 still upsets the angular dependence quite severely compared to the ordinary commutative case.

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on the photon mass.[11]

If we take the trace-U(1) as an additional (to the photon) gauge boson from the unbroken subgroup H, we would still get strong Lorentz symmetry violation since the trace-U(1) is not completely decoupled.

In summary, we found in this section that additional trace-U(1) subgroups are not completely decoupled and should lead to observable effects. In particular, if SUSY is not exact we have non-vanishing Π_2 which gives rise to strong Lorentz symmetry violation which has not been observed. This rules out possibilities (a) and (c) of section 1. Moreover, we confirmed that a trace-U(1) is not suitable as a photon candidate.

3. Mixing of trace and traceless parts

From the previous section we concluded that the trace-U(1) groups are unviable as candidates for the SM photon. Therefore, it has been suggested to construct the photon from traceless U(1) subgroups [8]. It turns out, however, that typically trace and traceless parts mix and the trace parts contribute their Lorentz symmetry violating properties to the mixed particle.

For U(2) broken by a fundamental Higgs, the standard Higgs mechanism yields the symmetry breaking $U(2) \to U(1)$. However, the remaining U(1) is a mixture of trace and traceless parts. If SUSY is broken, the trace-U(1) has a Π_2 part in the polarisation tensor. Taking this into account we find the following matrix for the equations of motion

$$\begin{pmatrix} \Pi_1^{\mathrm{U}(1)}k^2-\Pi_2-m^2 & m^2 & & & & & & \\ m^2 & \Pi_1^{\mathrm{SU}(2)}k^2-m^2 & & & & & & \\ & & \Pi_1^{\mathrm{U}(1)}k^2-m^2 & m^2 & & & & \\ & & m^2 & \Pi_1^{\mathrm{SU}(2)}k^2-m^2 & & & & \\ & & & & \Pi_1^{\mathrm{U}(1)}k^2-m^2 & m^2 & & \\ & & & & m^2 & \Pi_1^{\mathrm{SU}(2)}k^2-m^2 & & \\ & & & & & & \ddots & \end{pmatrix}, \tag{3.1}$$

where the adjoint U(2) and polarisation indices are $(0,1),(3,1),(0,2),(3,2),(0,3),(3,3)$. We omitted the values 1 and 2 for the adjoint U(2) indices which do not mix with the trace-U(1) and are not qualitatively different from the commutative case.

The matrix is block diagonal and the second and third polarisation (lower right corner) behave more or less like their commutative counterparts. We can concentrate on the upper left 2×2 matrix corresponding to the transverse polarisations affected by Π_2.

This 2×2 matrix admits two solutions for the equations of motion. Expanding for small Π_2 we find,

$$\left(\Pi_1^{\mathrm{U}(1)} + \Pi_1^{\mathrm{SU}(N)}\right) k^2 = \Pi_2 + O(\Pi_2^2)\,, \tag{3.2}$$

$$\left(\Pi_1^{\mathrm{U}(1)} + \Pi_1^{\mathrm{SU}(N)}\right) k^2 = \frac{\left(\Pi_1^{\mathrm{U}(1)} + \Pi_1^{\mathrm{SU}(N)}\right)^2}{\Pi_1^{\mathrm{U}(1)}\Pi_1^{\mathrm{SU}(N)}} m^2 + \frac{\Pi_1^{\mathrm{SU}(N)}}{\Pi_1^{\mathrm{U}(1)}}\Pi_2 + O(\Pi_2^2),$$

[11] Even fine-tuning of (2.20) to zero is not an option. Since we have only a finite number of masses this is at best possible for a finite number of values of $|\tilde{k}|$ and we will surely find values of $|\tilde{k}|$ where Π_2 is nonzero.

in analogy to (2.27). In absence of Π_2 the first solution in eq. (3.2) is a massless one corresponding to the massless combination of gauge bosons (think of it as the photon). The second is a massive combination (similar to the Z boson). The presence of non-vanishing Π_2 again leads to a mass $\frac{\Pi_2}{\Pi^{U(1)}}$ for the first solution and rules out the "massless" combination as a reasonable photon candidate.

This example demonstrates that the disastrous effects of Π_2 are also present in any combination which has an admixture of trace-U(1) degrees of freedom. Hence, this rules out possibilities (b) and (c) from the introduction.

4. Trace-U(1) factors in the unbroken subgroup

In the previous section, we learned in a specific example that even a small admixture of a trace part spoils the masslessness of the gauge boson corresponding to the unbroken gauge symmetry. This shows that a viable photon candidate must have a generator with vanishing (small is not enough) trace.

In our U(2) example with the gauge symmetry broken by a fundamental Higgs field the trace does not vanish. The generator corresponding to the unbroken U(1) is

$$\sim \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{4.1}$$

which obviously has non-vanishing trace.

One can try to construct other symmetry breaking mechanisms with larger groups and products of groups as well as the other representations for the Higgs fields allowed by the condition 3 of the introduction. However, one always encounters one of the following situations. Either the remaining U(1) has a generator with non-vanishing trace or there is more than one unbroken U(1) subgroup. Both situations are in contradiction of observations, as our discussion of the previous sections shows.

This is generalised and more precisely formulated by the following proposition (already stated in the introduction): *Consider a scenario where a set of fundamental, bifundamental and adjoint Higgs fields breaks $U(N_1) \times U(N_2) \times \cdots \times U(N_m) \to H$, such that H is non-trivial. Then there is at least one generator of the unbroken subgroup H with non-vanishing trace. This generator can be chosen such that it generates a U(1) subgroup.*

Let us now turn to a proof of the proposition. Let us start with the simple situation of one U(N) group. Since we have only one group, we have only fundamental and adjoint Higgs fields at our disposal. We proceed by switching on one Higgs field (component) after the other. Let us start with the fundamental field. U(N) symmetry allows us to chose this field as

$$\phi_{\mathrm{f}} = (0, \ldots, 0, a)^{\mathrm{T}}. \tag{4.2}$$

Case 1: If $a = 0$ we have no breaking with a fundamental Higgs. In this case we are finished, because the generator of the original trace-U(1) is proportional to the $N \times N$ unit matrix and therefore commutes with any adjoint Higgs field. Therefore this generator continues to generate an unbroken trace-U(1) subgroup, as stated in the proposition.

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Case 2: If $a \neq 0$ gauge symmetry is broken down to the $\mathrm{U}(N-1)$ living in the upper $N-1$ components of any field. A set of generators for this group are the ordinary $\mathrm{U}(N-1)$ in the upper left $(N-1)\times(N-1)$ submatrix and zero in the other components. In particular, there is a new trace-U(1) with generator

$$T^1_{\mathrm{trace}} = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & 0 \end{pmatrix}. \tag{4.3}$$

Under this subgroup an adjoint field decomposes into

$$\phi_{\mathrm{ad}} = \left(\begin{array}{c|c} \phi^2_{\mathrm{ad}} & \phi^2_{\mathrm{f}} \\ \hline (\phi^2_{\mathrm{f}})^\dagger & \phi^2_{\mathrm{s}} \end{array}\right), \tag{4.4}$$

where ϕ^2_{ad}, ϕ^2_{f} and ϕ^2_{s} are adjoint, fundamental and singlett fields under the remaining $\mathrm{U}(N-1)$ symmetry. An additional fundamental field $\hat{\phi}_{\mathrm{f}}$ decomposes as

$$\hat{\phi}_{\mathrm{f}} = \begin{pmatrix} \hat{\phi}^2_{\mathrm{f}} \\ \hat{\phi}^2_{\mathrm{s}} \end{pmatrix} \tag{4.5}$$

into an additional fundamental $\hat{\phi}^2_{\mathrm{f}}$ and another singlett $\hat{\phi}^2_{\mathrm{s}}$. We can now repeat the argument for the remaining $\mathrm{U}(N-1)$ group starting, again, with the fundamental fields.
This procedure has to stop at some point, i.e. at one point the fundamental ϕ^n_{f} has to be zero, or the symmetry is broken completely and H would be the trivial group in violation of the assumptions.

For a product of more than one group the proof is analogous only that we have additional bifundamental fields. Let us briefly consider the situation with a product of two groups $\mathrm{U}(M)\times\mathrm{U}(N)$. Switching on fundamental fields we can end up with:
Case 1: If all fundamentals are zero the symmetry remains unbroken $\mathrm{U}(M)\times\mathrm{U}(N)$. One can easily see that bifundamental and adjoint fields cannot break the trace-U(1) generated by the $(N+M)\times(N+M)$ unit matrix.[12]
Case 2: Let us switch on one fundamental field. Without loss of generality we can take it to be an N fundamental. The symmetry is broken down to $\mathrm{U}(M)\times\mathrm{U}(N-1)$ with a new trace-U(1) for the $\mathrm{U}(N-1)$ in analogy to the simple $\mathrm{U}(N)$ situation discussed above. All fields transforming under the $\mathrm{U}(M)$ remain unaffected. The fundamental and adjoint fields for $\mathrm{U}(N)$ are decomposed according to eqs. (4.4), (4.5). Finally the bifundamental decomposes as

$$\phi_b = \left(\begin{array}{c|c} \phi^2_{\mathrm{b}} & \phi^2_{\mathrm{b,f}} \end{array}\right) \tag{4.6}$$

into a bifundamental ϕ^2_{b} under $\mathrm{U}(M)\times\mathrm{U}(N-1)$ and a fundamental $\phi^2_{\mathrm{b,f}}$ under $\mathrm{U}(N-1)$. The argument proceeds by induction. The case of more than two $\mathrm{U}(N)$ factors is completely analogous.

[12] We can think of $\mathrm{U}(M)\times\mathrm{U}(N)$ embedded into $\mathrm{U}(N+M)\times\mathrm{U}(N+M)$

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5. Conclusions

Noncommutative gauge symmetry in the Weyl-Moyal approach leads to two main features which have to be taken into account for sensible model building. First, there are strong constraints on the dynamics and the field content. The only allowed gauge groups are U(N). In addition, the matter fields are restricted to transform as fundamental, bifundamental and adjoint representations of the gauge group. Finally, anomaly freedom for noncommutative theories requires the theory to be vector like.[13] Second, there are the effects of ultraviolet/infrared mixing. Those lead to asymptotic infrared freedom of the trace-U(1) subgroup and, if the model does not have unbroken supersymmetry, to Lorentz symmetry violating terms in the polarisation tensor for this trace-U(1) subgroup.

We have demonstrated that, although the trace-U(1) decouples in the limit $k \to 0$, the coupling is not negligibly small at finite momentum scales k, as they appear, for example, in scattering experiments. Therefore, observations rule out additional unbroken (massless) trace-U(1) subgroups. An example is the model considered in ref. [8]. In ref. [8], the trace-U(1) groups were completely discarded before the symmetry breaking scheme was discussed. A more careful investigation which takes takes into account these subgroups yields the symmetry breaking $\mathrm{U}(4) \times \mathrm{U}(3) \times \mathrm{U}(2) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times (\mathrm{U}(1))^4$ instead of $\mathrm{U}(4) \times \mathrm{U}(3) \times \mathrm{U}(2) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. Therefore we have superfluous U(1) subgroups. Following the above lines explicitely one easily finds that one of the U(1)'s has a generator which is proportional to the 9×9 unity matrix.

Noncommutativity explicitly breaks Lorentz invariance. Therefore an additional Lorentz symmetry violating structure is allowed in the polarisation tensor. This structure is absent only in supersymmetric models. If supersymmetry is (softly) broken, this additional structure is present in the polarisation tensor of the trace-U(1). It leads to an additional mass $\sim \Delta M^2_{\mathrm{SUSY}}$ for one of the transverse polarisation states [33]. The tight constraints on the photon mass therefore exclude trace-U(1)'s as a candidate for the photon. It turns out that even a small admixture of a trace part to a traceless part (unaffected by these problems) is fatal. The only way out seems to be the construction of the photon from a completely traceless generator. A group theoretic argument shows, that this is impossible whithout having additional unbroken U(1) subgroups. However, those are already excluded from the arguments given above.

This result severely restricts the possibilities to construct a noncommutative Standard model extension. If all of the constraints given at the beginning are fulfilled the noncommutativity scale is pushed to scales far beyond M_{P}. This is to be compared to the less restrictive constraints $M_{\mathrm{NC}} \gtrsim 0.1 - 10\,\mathrm{TeV}$ (conservative estimate) obtained from tree level amplitudes [36] or from an approach where a Taylor expansion in the noncommutativity parameters is used before quantization, thereby ignoring effects of ultraviolet/infrared mixing and possibly constraints on the field content [7, 6, 26 – 32]. We stress, however, that

[13] In turn, this eliminates the Green-Schwarz mechanism [34] as a possible source for a (large) mass term for the trace-U(1) part of the gauge group.

the latter approach may lead to a completely different quantum theory and therefore our bounds may not be applicable.

We would like to conclude with a more optimistic prospect.

In general there is no reason to assume that the simple noncommutative model used here describes correctly the physics at energies ranging from a few eV up to the Planck mass. In fact, due to the ultraviolet/infrared mixing, a different ultraviolet embedding of the theory would modify the theory not only in the ultraviolet, but also in the infrared which can drastically alter our conclusions [23]. In particular, our conclusions are tied to a slow logarithmic decoupling of the trace-U(1), but if it is changed to a power-like decoupling, the U(1) factors would safely decouple and leave the Standard Model in peace. We expect that this can be achieved by embedding the noncommutative theory into a higher dimensional theory in the ultraviolet (which will have a power-like beta function) and then appeal to the ultraviolet/infrared mixing to transport this power-like behaviour to the infrared region for the trace-U(1) gauge coupling (see later work [23]).

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A. Polarsation directions in gauge theories

In this section we review some basics about the counting of degrees of freedom in gauge theories. In particular, we show how gauge invariance reduces the number of degrees of freedom from the naive 4 (4 components of the vector field) to 2 and 3 for the massless and massive case, respectively.

A.1 The massless case

In ordinary QED, the field equations read

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0. \tag{A.1}$$

Using Lorentz gauge,

$$\partial_\mu A^\mu = 0 \tag{A.2}$$

eq. (A.1) simplifies to the wave equation

$$\Box A^\mu = 0. \tag{A.3}$$

Writing

$$A^\mu = C\epsilon^\mu \exp(ikx), \tag{A.4}$$

any ϵ^μ is a solution to (A.3) as long as

$$k^2 = 0. \tag{A.5}$$

So far we have all 4 polarisations. However, (A.2) implies 4-dimensional transversality,

$$k_\mu \epsilon^\mu = 0, \tag{A.6}$$

and reduces the allowed number of polarisations to three. This is still more than the two polarisation states a photon should have.

However, Lorentz gauge does not completely fix the gauge. We can still use a gauge transformation Ω with $\Box\Omega = 0$. This allows us to choose $A^0 = 0$. Together with (A.6) this leads us to the ordinary 3-dimensional transversality,

$$\overrightarrow{k} \cdot \overrightarrow{\epsilon} = 0. \tag{A.7}$$

A.2 The case with a Higgs field

The presence of a Higgs field modifies (A.1),

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + m^2 A^\mu + m\partial^\mu \phi_2 = 0. \tag{A.8}$$

Moreover it supplies an additional equation for the Goldstone boson ϕ_2,

$$\Box\phi_2 + m\partial_\mu A^\mu = 0. \tag{A.9}$$

One convenient choice of gauge is unitary gauge where

$$\phi_2 = 0. \tag{A.10}$$

We stress from the beginning that unitary gauge implies (A.2), as can be seen from (A.9). In this gauge eq. (A.8) simplifies,

$$\Box A^\mu + m^2 A^\mu = 0. \tag{A.11}$$

Now everything runs in a similar fashion to the massless case, only that

$$k^2 - m^2 = 0. \tag{A.12}$$

The important difference is that unitary gauge fixes the gauge completely. We cannot make an additional gauge choice. Therefore it is impossible to get rid of the 3rd polarisation state which satisfies Lorentz gauge $k_\mu \epsilon^\mu = 0$. Stated differently we cannot require 3-dimensional transversality for ϵ^μ and we have therefore three allowed polarisation states with equal masses.

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